Mergers of Stellar-Mass Black Holes in Nuclear Star Clusters

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ABSTRACT

Mergers between stellar-mass black holes will be key sources of gravitational radiation for ground-based detectors. However, the rates of these events are highly uncertain, given that such systems are invisible. One formation scenario involves mergers in field binaries, where our lack of complete understanding of common envelopes and the distribution of supernova kicks has led to rate estimates that range over a factor of several hundred. A different, and highly promising, channel involves multiple encounters of binaries in globular clusters or young star clusters. However, we currently lack solid evidence for black holes in almost all such clusters, and their low escape speeds raise the possibility that most are ejected because of supernova recoil. Here we propose that a robust environment for mergers could be the nuclear star clusters found in the centers of small galaxies. These clusters have millions of stars, black hole relaxation times well under a Hubble time, and escape speeds that are several times those of globulars, hence they retain most of their black holes. We present simulations of the three-body dynamics of black holes in this environment and estimate that, if most nuclear star clusters do not have supermassive black holes that interfere with the mergers, at least several tens of events per year will be detectable with Advanced LIGO.

Subject headings: black hole physics – galaxies: nuclei – gravitational waves — relativity

1. Introduction

Ground-based gravitational wave detectors have now achieved their initial sensitivity goals (e.g., Abbott et al. 2007). In the next few years, these sensitivities are expected to improve by a factor of ~ 10, which will increase the searchable volume by a factor of $\sim 10^3$ and will lead to many detections per year.

One of the most intriguing possible sources for such detectors is the coalescence of a double stellar-mass black hole binary. Such binaries are inherently invisible, meaning that

we have no direct observational handle on how common they are or their masses, spin magnitudes, or orientations. Comparison of the observed waveforms (or of waveforms from merging supermassive black holes) with predictions based on post-Newtonian analysis and numerical relativity will be the most direct possible test of the predictions of strong-gravity general relativity.

The electromagnetic non-detection of these sources makes rate estimates highly challenging, because our only observational handles on BH-BH binaries come from possible progenitors. For example, a common scenario involves the effectively isolated evolution of a field binary containing two massive stars into a binary with two black holes that will eventually merge (e.g., Lipunov et al. 1997; Belczynski & Bulik 1999). There are profound uncertainties involved in calculations of these rates due to e.g., the lack of knowledge of the details of the common envelope phase in these systems and the absence of guides to the distribution of supernova kicks delivered to black holes. As a recent indication of the range of estimated rates, note that the Advanced LIGO detection rate of BH-BH coalescences is estimated to be anywhere between $\sim 1 - 500$ yr^{-1} by Belczynski et al. (2007), depending on how common envelopes are modeled.

Another promising location for BH-BH mergers is globular clusters or super star clusters, where stellar number densities are high enough to cause multiple encounters and hardening of binaries. Even though binaries are kicked out before they merge (Kulkarni, Hut, & McMillan 1993; Sigurdsson & Hernquist 1993; Sigurdsson & Phinney 1993, 1995; Portegies Zwart & McMillan 2000; O'Leary et al. 2006), these clusters can still serve as breeding grounds for gravitational wave sources. Indeed, O'Leary, O'Shaughnessy, & Rasio (2007) estimate a rate of 0.5 yr^{-1} for initial LIGO and 500 yr^{-1} for Advanced LIGO via this channel. There is, however, little direct evidence for black holes in most globulars (albeit they could be difficult to see). In addition, at least one black hole in a low-mass X-ray binary apparently received a $\gtrsim 100$ km s^{-1} kick from its supernova (GRO J1655–40; see Mirabel et al. 2002). This is double the escape speed from the centers of even fairly rich globulars (Webbink 1985), leading to uncertainties about their initial black hole population and current merger rates.

Here we propose that mergers occur frequently in the nuclear star clusters that may be in the centers of many low-mass galaxies (Böker et al. 2002; Ferrarese et al. 2006; Wehner & Harris 2006; note that some of these are based on small deviations from smooth surface brightness profiles and are thus still under discussion). It has recently been recognized that in these galaxies, which may not have supermassive black holes (for a status report on ongoing searches for low-mass central black holes, see Greene & Ho 2007), the nuclear clusters have masses that are correlated with the surrounding velocity dispersion σ as $M \approx 10^7\ M_\odot(\sigma/54\ \mathrm{km\ s^{-1}})^{4.3}$ (Ferrarese et al. 2006). When the velocity dispersion is in

the range of $\sim 30 - 60$ km s^{-1}, the half-mass relaxation time is small enough that black holes (which have $\sim 20\times$ the average stellar mass) can sink to the center in much less than a Hubble time. In addition, although systems with equal-mass objects require roughly 15 half-mass relaxation times to undergo core collapse (Binney & Tremaine 1987), studies show that systems with a wide range of stellar masses experience core collapse within $\sim 0.2\times$ the half-mass relaxation time (Portegies Zwart & McMillan 2002; Gürkan et al. 2004). Therefore, clusters with masses less than $\sim$ few $\times 10^7\ M_\odot$ will have collapsed by now and hence increased the escape speed from the center, allowing retention of most of their black holes.

As we show in this paper, nuclear star clusters are therefore excellent candidates for stellar-mass black hole binary mergers because they keep their black holes while also evolving rapidly enough that the holes can sink to a region of high density. If tens of percent of the black holes in eligible galaxies undergo such mergers, the resulting rate for Advanced LIGO is at least several tens per year. In § 2 we quantify these statements and results more precisely and discuss our numerical three-body method. We give our conclusions in § 3.

2. Method and Results

2.1. Characteristic times and initial setup

Our approach is similar to that of O'Leary et al. (2006), who focus on globular clusters with velocity dispersions $\sigma \leq 20$ km s^{-1}. Here, however, we concentrate on the more massive and tightly bound nuclear star clusters. Our departure point is the relation found by Ferrarese et al. (2006) between the masses and velocity dispersions of such clusters:

$$M_{\rm nuc} = 10^{6.91\pm0.11} \left(\sigma/54\ {\rm km\ s^{-1}}\right)^{4.27\pm0.61}\ M_\odot\ . \tag{1}$$

Assuming that there is no massive central black hole for these low velocity dispersions, the half-mass relaxation time for the system is (see Binney & Tremaine 1987) $t_{\rm rlx} \approx \frac{N/2}{8\ln N} t_{\rm cross}$ where $N \approx M_{\rm nuc}/0.5\ M_\odot$ is the number of stars in the system (assuming an average mass of 0.5 $M_\odot$) and $t_{\rm cross} \approx R/\sigma$ is the crossing time. Here $R = GM_{\rm nuc}/\sigma^2$ is the radius of the cluster. Putting this together gives

$$t_{\rm rlx} \approx 1.3 \times 10^{10}\ {\rm yr} \left(\sigma/54\ {\rm km\ s^{-1}}\right)^{5.54\pm0.61}\ . \tag{2}$$

The relaxation time scales inversely with the mass of an individual star (Binney & Tremaine 1987), so a 10 $M_\odot$ black hole will settle in roughly 1/20 of this time. Also note that large N-body simulations with broad mass functions evolve to core collapse within roughly 0.2 half-mass relaxation times (Portegies Zwart & McMillan 2002; Gürkan et al. 2004), hence in

the current universe clusters with velocity dispersions $\sigma < 60$ km s^{-1} will have had their central potentials deepened significantly.

The amount of deepening of the potential, and thus the escape speed from the center of the cluster, depends on uncertain details such as the initial radial dependence of the density and the binary fraction. Given that the timescale for segregation of the black holes in the center is much less than a Hubble time, we will assume that the escape speed is roughly 5× the velocity dispersion, as is the case for relatively rich globular clusters (Webbink 1985). This may well be somewhat conservative, because the higher velocity dispersion here than in globulars suggests that a larger fraction of binaries will be destroyed in nuclear star clusters. This could lead to less efficient central energy production and hence deeper core collapse than is typical in globulars.

With this setup, our task is to follow the interactions of black holes in the central regions of nuclear star clusters, where we will scale by stellar number densities of $n \sim 10^6$ pc^{-3} because of density enhancements caused by relaxation and mass segregation. Some black holes will begin their lives in binaries, but to be conservative we will assume that they start as single objects and have to exchange into binaries that contain main sequence stars or other objects. All binaries in the cluster will be hard, i.e., will have internal energies greater than the average kinetic energy of a field star, because otherwise they will be softened and ionized quickly (e.g., Binney & Tremaine 1987). If, for example, we consider binaries of two 1 $M_\odot$ stars in a system with $\sigma = 50$ km s^{-1}, this means that the semimajor axis has to be less than $a_{\max} \sim 1$ AU. Studies of main sequence binaries in globular clusters, which have $\sigma \sim 10$ km s^{-1}, suggest that after billions of years roughly 5–20% of them survive, with the rest falling victim to ionization or collisions (Ivanova et al. 2005). The binary fraction will be lower in nuclear star clusters due to their enhanced velocity dispersion, but since when binaries are born they appear to have a constant distribution across the log of the semimajor axis from $\sim 10^{-2} - 10^3$ AU (e.g., Abt 1983; Duquennoy & Mayor 1991) the reduction is not necessarily by a large factor. We will conservatively scale by a binary fraction $f_{\rm bin} = 0.01$.

If a black hole with mass $M_{\rm BH}$ gets within a couple of semimajor axes of a main sequence binary, the binary will tidally separate and the BH will acquire a companion. The timescale on which this happens is $t_{\rm bin} = (n\Sigma\sigma)^{-1}$, where $\Sigma = \pi r_p^2\,(1 + 2GM_{\rm tot}/(\sigma^2 r_p))$ is the interaction cross section for pericenter distances $\leq r_p$ when gravitational focusing is included. Here $M_{\rm tot}$ is the mass of the black hole plus the mass of the binary. If we assume that $M_{\rm BH} = 10\ M_\odot$ and it interacts with a binary with two 1 $M_\odot$ members and an $a = 1$ AU semimajor axis, then the typical timescale on which a three-body interaction and capture of one of the stars occurs is

$$t_{3-{\rm bod}} = (n\Sigma\sigma)^{-1} \approx 1.2 \times 10^9\ {\rm yr}(n/10^6\ {\rm pc}^{-3})^{-1}(f_{\rm bin}/0.01)^{-1}(\sigma/50\ {\rm km\ s})(a/1\ {\rm AU})^{-1}\ . \quad (3)$$

This is small enough compared to a Hubble time that we start our simulations by assuming that each black hole has exchanged into a hard binary, and follow its evolution from there.

Another important question is whether, after a three-body interaction, a black hole binary will shed the kinetic energy of its center of mass via dynamical friction and sink to the center of the cluster before another three-body encounter. If not, the kick speeds will add in a random walk, thus increasing the ejection fraction.

To compute this we note that the local relaxation time of a binary is

$$t_{\rm rlx} = \frac{0.339}{\ln\Lambda} \frac{\sigma^3}{G^2 \langle m \rangle M_{\rm bin} n} \tag{4}$$

(Spitzer 1987) where σ is the local velocity dispersion, $\ln\Lambda \sim 10$ is the Coulomb logarithm, $\langle m \rangle$ is the average mass of interloping stars, n is their number density, and $M_{\rm bin}$ is the mass of the binary. The timescale for a three-body interaction is $t_{3-{\rm bod}} = (n\Sigma\sigma)^{-1}$ as above. For a gravitationally focused binary, which is of greatest interest because only these could in principle produce three-body recoil sufficient to eject binaries or singles, $r_p < GM_{\rm bin}/\sigma^2$. Therefore, $\Sigma \approx 2\pi r_p GM_{\rm bin}/\sigma^2$ and

$$t_{3-{\rm bod}} \approx \frac{\sigma}{2\pi n r_p G M_{\rm bin}}\ . \tag{5}$$

If we let $r_p = qGM_{\rm bin}/\sigma^2$, with $q < 1$, then

$$t_{3-{\rm bod}} \approx \frac{\sigma^3}{2\pi q G^2 M_{\rm bin}^2 n} \tag{6}$$

so that

$$t_{\rm rlx}/t_{3-{\rm bod}} \approx \frac{2q}{\ln\Lambda} \frac{M_{\rm bin}}{\langle m \rangle}\ . \tag{7}$$

In the center of a cluster, where mass segregation is likely to have flattened the mass distribution, we find that this quantity is typically less than unity (and it decreases as the binary hardens), meaning that after a three-body encounter a binary has an opportunity to share its excess kinetic energy via two-body encounters and thus settle back to the center of the cluster. We therefore treat the encounters separately rather than adding the kick speeds in a random walk.

In a given encounter, suppose that a binary of total mass $M_{\rm bin} = M_1 + M_2$, a reduced mass $\mu = M_1 M_2/M_{\rm bin}$, and a semimajor axis $a_{\rm init}$ interacts with an interloper of mass $m_{\rm int}$, and that the kinetic energy of the interloper at infinity is much less than the binding energy of the binary (i.e., this is a very hard interaction). If after the interaction the semimajor axis is $a_{\rm fin} < a_{\rm init}$, then energy and momentum conservation mean that the recoil speed of the

binary is given by $v_{\rm bin}^2 = G\mu \frac{m_{\rm int}}{M_{\rm bin}+m_{\rm int}}(1/a_{\rm fin} - 1/a_{\rm init})$, and the recoil speed of the interloper is $v_{\rm int} = (M_{\rm bin}/m_{\rm int})v_{\rm bin}$. For example, suppose that $M_1 = M_2 = 10\ M_\odot$, $M_{\rm int} = 1\ M_\odot$, $a_{\rm init} = 0.1$ AU, and $a_{\rm fin} = 0.09$ AU. The binary then recoils at $v_{\rm bin} = 15$ km s^{-1} and stays in the cluster, whereas the interloper recoils at $v_{\rm int} = 300$ km s^{-1} and is ejected.

2.2. Results

The central regions of the clusters undergo significant mass segregation, and thus the mass function will be at least flattened, and possibly inverted. This has, for example, been observed for globulars (Sosin 1997). To include this effect, when we consider the mass of a black hole, its companion, or the interloping third object in a binary-single encounter, we go through two steps. First we select a zero age main sequence (ZAMS) mass between 0.2 $M_\odot$ and 100 $M_\odot$ using a simple power law distribution $dN/dM \propto M^{-\alpha}$. We allow α to range anywhere from 2.35 (the unmodified Salpeter distribution) to -1.0, where smaller values indicate the effects of mass segregation. Second, we evolve the ZAMS mass to a current mass. Our mapping is that for $M_{\rm ZAMS} < 1\ M_\odot$, the star is still on the main sequence and retains its original mass; for $1\ M_\odot < M_{\rm ZAMS} < 8\ M_\odot$ the star has evolved to a white dwarf, with mass $M_{\rm WD} = 0.6\ M_\odot + 0.4\ M_\odot(M_{\rm ZAMS}/M_\odot - 0.6)^{1/3}$; for $8\ M_\odot < M_{\rm ZAMS} < 25\ M_\odot$ the star has evolved to a neutron star, with mass $M_{\rm NS} = 1.5\ M_\odot + 0.5\ M_\odot(M_{\rm ZAMS} - 8\ M_\odot)/17\ M_\odot$; and for $M_{\rm ZAMS} > 25\ M_\odot$ the star has evolved to a black hole with mass $M_{\rm BH} = 3\ M_\odot + 17\ M_\odot(M_{\rm ZAMS} - 25\ M_\odot)/75\ M_\odot$. Therefore, we assume that black hole masses range from 3 $M_\odot$ to 20 $M_\odot$.

These prescriptions are overly simplified in many ways. We therefore explore different mass function slopes, main sequence cutoffs, and so on, and find that our general picture is robust against specific assumptions. Note that, consistent with O'Leary et al. (2006), we find that there is a strong tendency for the merged black holes to be biased towards high masses. Therefore, if black holes with masses $> 20\ M_\odot$ are common, these will dominate the merger rates. This is important for data analysis strategies, because the low-frequency cutoff of ground-based gravitational wave detectors implies that higher-mass black holes will have proportionally more of their signal in the late inspiral, merger, and ringdown.

The three-body interactions themselves are assumed to be Newtonian interactions between point masses and are computed using the hierarchical N-body code HNBody (K. Rauch and D. Hamilton, in preparation), using the driver IABL developed by Kayhan Gültekin (see Gültekin, Miller, & Hamilton 2004, 2006 for a detailed description). These codes use a number of high-accuracy techniques to follow the evolution of gravitating point masses. Between interactions, we use the Peters equations (Peters 1964) to follow the gradual inspiral and

circularization of the binary via emission of gravitational radiation. This is negligible except near the end of any given evolution.

We begin by selecting the mass of the black hole and of its companion (which does not need to be a black hole) from the evolved mass function. We also begin with a semimajor axis that is 1/4 of the value needed to ensure that the binary is hard. We do this because soft binaries are likely to be ionized and thus become single stars rather than merge. We also select an eccentricity from a thermal distribution $P(e)de = 2ede$. We then allow the binary to interact with single field stars drawn from the evolved mass function, one at a time, until either (1) the binary merges due to gravitational radiation, (2) the binary is split apart and thus ionized (this is exceedingly rare given our initial conditions), or (3) the binary is ejected from the cluster. The entire set of interactions until merger typically takes millions to tens of millions of years, and only rarely over a hundred million years, so it finishes in much less than a Hubble time. In the course of these interactions there are typically a number of exchanges, which usually swap in more massive for less massive members of the binary. This is the cause of the bias towards high-mass mergers that was also found by O'Leary et al. (2006). As shown in Table 1, for $\alpha < 1$ most black holes acquire a black hole companion in the process of exchanges, and for $\alpha \leq 0.5$ virtually all do.

The results in Table 1 are focused on different mass function slopes and escape speeds. As expected, we find that for $V_{\rm esc} > 150$ km s^{-1} the overwhelming majority of black hole binaries merge in the nuclear star cluster rather than being ejected (see Figure 1). This is the difference from lower-σ globular clusters, where the mergers happen outside the cluster. Note also that in addition to few binaries being ejected, there are typically only 1–2 single black holes ejected per merger, suggesting that $> 50\%$ of holes will merge. In contrast, at the 50 km s^{-1} escape speed typical of globulars, > 20 single black holes are ejected per merger, suggesting an efficiency of $< 10\%$. For well-segregated clusters (with $\alpha \leq 0$), the average mass of black holes that merge, binary ejection fraction and number of singles ejected, and number of black holes that merge with each other instead of other objects are all insensitive to the particular mass function slope. For less segregated clusters with $\alpha > 0$, the retention fraction of black holes rises rapidly to unity because most of the objects that interact with the holes are less massive stars. In such cases there might be a channel by which the mass of the holes increases via accretion of stars, but we expect $\alpha > 0$ to be rare for nuclear star clusters because of the shortness of the segregation times of black holes. Overall, there appears to be a wide range of realistic parameters in which fewer than 10% of binary black holes are ejected before merging.

3. Discussion and Conclusions

We have shown that nuclear star clusters with velocity dispersions around $\sigma \sim 30-60$ km s^{-1} are promising breeding grounds for stellar-mass black hole mergers. At significantly lower velocity dispersions, as found in globulars, the escape speed is low enough that the binaries are ejected before they merge. Significantly higher velocity dispersions appear correlated with the appearance of supermassive black holes (Gebhardt et al. 2000; Ferrarese & Merritt 2000). In such an environment there might also be interesting rates of black hole mergers, but the increasing velocity dispersion closer to the central object means that binary fractions are lower and softening, ionization, or tidal separation by the supermassive black hole itself are strong possibilities for stellar-mass binaries (Miller et al. 2005; Lauburg & Miller, in preparation).

To estimate the rate of detections with Advanced LIGO, we note that velocity dispersions in the $\sigma \sim 30-60$ km s^{-1} range correspond to roughly a factor of ~ 10 in galaxy luminosity (Ferrarese et al. 2006). Galaxy surveys suggest (e.g., Blanton et al. 2003) that for dim galaxies the luminosity function scales as roughly $dN/dL = \phi^*(L/L_*)^{-\alpha}$, where $\phi^* = 1.5 \times 10^{-2}h^3$ Mpc^{-3} $\approx 5 \times 10^{-3}$ Mpc^{-3} for $h = 0.71$, and $\alpha \approx -1$. This implies that there are nearly equal numbers of galaxies in equal logarithmic bins of luminosity. A factor of 10 in luminosity is roughly e^2, so the number density of relevant galaxies is approximately 10^{-2} Mpc^{-3}. To get the rate per galaxy, we note that typical initial mass functions and estimates of the mass needed to evolve into a black hole combine to suggest that for a cluster of mass $M_{\rm nuc}$, approximately $3 \times 10^{-3}(M_{\rm nuc}/M_\odot)$ stars evolve into black holes (O'Leary, O'Shaughnessy, & Rasio 2007). That implies a few$\times 10^4$ black holes per nuclear star cluster. If a few tens of percent of these merge in a Hubble time, and if the rate is slightly lower now because many of the original black holes have already merged (see O'Leary et al. 2006), that suggests a merger rate of $> 0.1 \times {\rm few} \times 10^4/(10^{10}$ yr) per galaxy, or few$\times 10^{-9}$ Mpc^{-3} yr^{-1}. At the ~ 2 Gpc distance at which Advanced LIGO is expected to be able to see mergers of two 10 $M_\odot$ black holes (see, e.g., Mandel 2007), the available volume is 3×10^{10} Mpc^3, for a rate of $\gtrsim 100$ per year. Roughly 50–80% of galaxies in the eligible luminosity range appear to have nuclear star clusters (see Ferrarese et al. 2006 for a summary). If the majority of the clusters do not have a supermassive black hole, this suggests a final rate of at least several tens per year for Advanced LIGO. This could be augmented somewhat by small galaxies that originally had supermassive black holes, but had them ejected after a merger and then reformed a central cluster (Volonteri 2007; Volonteri, Haardt, & Gültekin 2008).

For nearby ($z < 0.1$) events of this type it might be possible to identify the host galaxy. However, for more typical $z \sim 0.5 \Rightarrow d \approx 2$ Gpc events the number of can-

didates is too large: even assuming angular localization of $\Delta\Omega = (1^\circ)^2$ and a distance accuracy of $\Delta d/d = 1\%$, the number of galaxies in the right luminosity range is $N \sim 4\pi/3(2000\ \mathrm{Mpc})^3(\Delta\Omega/4\pi)(\Delta d/d)(0.01\ \mathrm{Mpc}^{-3}) \approx 80$. Therefore, barring some unforseen electromagnetic counterpart, the host will usually not be obvious.

We anticipate that tens per year is a somewhat conservative number, because our simulations suggest that more like 50% of black holes will be retained, even as single objects, and because (unlike in a globular cluster) the central regions of galaxies are not devoid of gas, hence more black holes could form in the vicinity of the cluster and fall in. In addition, if stellar-mass black holes with masses beyond 20 $M_\odot$ are common, this also increases the detection radius and hence the rate. Even for total masses $\sim 30\ M_\odot$ and at redshifts $z \sim 0.5$, the observer frame gravitational wave frequency at the innermost stable circular orbit is $f_{\rm ISCO} \sim 4400\ \mathrm{Hz}/[30(1+z)] \sim 100$ Hz. This is close enough to the range where frequency sensitivity declines that detection of many of these events will rely strongly on the signal obtained from the last few orbits plus merger and ringdown. In much of this range, numerical relativity is essential.

As a final point, we note that for the same reason that nuclear star clusters are favorable environments for retention and mergers of stellar-mass black holes, they could also be good birthplaces for more massive black holes. This could be prevented, even for the relatively high escape speeds discussed here, if recoil from gravitational radiation during the coalescence exceeds ~ 200 km s^{-1}. The key uncertainty here is the spin magnitudes of the holes at birth. Numerous simulations demonstrate that high spins with significant projections in the binary orbital plane can produce kicks of up to several thousand kilometers per second (Gonzalez et al. 2007). If there is significant processing of gas through accretion disks the spins are aligned in a way that reduces the kick to below 200 km s^{-1} (Bogdanović, Reynolds, & Miller 2007), but stellar-mass black holes cannot pick up enough mass from the interstellar medium for this to be effective. For example, the Bondi-Hoyle accretion rate is $\dot{M}_{\rm Bondi} \approx 10^{-13}\ M_\odot\ \mathrm{yr}^{-1}(\sigma/50\ \mathrm{km\ s}^{-1})^{-3}(n/100\ \mathrm{cm}^{-3})(M/10\ M_\odot)^2$, meaning that to accrete the $\sim 1\%$ of the black hole mass needed to realign the spin (Bogdanović, Reynolds, & Miller 2007) would require at least a trillion years. Current estimates of stellar-mass black hole spins suggest $a/M > 0.5$ in many cases (Shafee et al. 2006; McClintock et al. 2006; Miller 2007; Liu et al. 2008). If the spins are isotropically oriented and uniformly distributed in the range $0 < a/M < 1$, and the mass ratios are in the $m_{\rm small}/m_{\rm big} \sim 0.6 - 0.8$ range typical in our simulations, then use of the Campanelli et al. (2007) or Baker et al. (2008) kick formulae imply that roughly 84% of the recoils exceed 200 km s^{-1} and 78% exceed 250 km s^{-1}. This suggests that multiple mergers are rare unless there is initially an extra-massive black hole as a seed (e.g., Holley-Bockelmann et al. 2008 for a discussion of the effects of gravitational wave recoil), but further study is important.

In conclusion, we show that the compact nuclear star clusters found in the centers of many small galaxies are ideal places to foster mergers between stellar-mass black holes. It is not clear whether multiple rounds of mergers can lead to runaway, but this is a new potential source for ground-based detectors such as Advanced LIGO, where numerical relativity will play an especially important role.

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Table 1. Simulations of nuclear star clusters[a]

$V_{\rm esc}$ (km s^{-1})[b]	$M_{\rm ms,max}$[c]	α[d]	$\langle M_{\rm BH}\rangle (M_\odot)$[e]	$f_{\rm merge}$[f]	$f_{\rm notBH}$[g]	$\langle M_{\rm bin,merge}\rangle (M_\odot)$[h]	$\langle N_{\rm single,eject}\rangle$[i]
50	$1M_\odot$	0	11.7	0.25	0.0	31.2	24.8
62.5	$1M_\odot$	0	11.7	0.33	0.0	31.6	15.3
75	$1M_\odot$	0	11.7	0.42	0.0	30.9	11.5
87.5	$1M_\odot$	0	11.7	0.52	0.0	31.9	7.9
20	$1M_\odot$	0	11.7	0.63	0.02	30.0	6.2
112.5	$1M_\odot$	0	11.7	0.68	0.0	31.4	4.7
125	$1M_\odot$	0	11.7	0.72	0.02	31.8	4.3
137.5	$1M_\odot$	0	11.7	0.76	0.01	32.0	3.0
150	$1M_\odot$	0	11.7	0.80	0.03	32.3	2.8
162.5	$1M_\odot$	0	11.7	0.93	0.03	31.3	2.0
175	$1M_\odot$	0	11.7	0.89	0.02	31.9	2.0
187.5	$1M_\odot$	0	11.7	0.90	0.01	31.3	2.1
200	$1M_\odot$	0	11.7	0.94	0.08	31.1	1.3
212.5	$1M_\odot$	0	11.7	0.89	0.05	30.5	1.0
225	$1M_\odot$	0	11.7	0.98	0.06	31.0	1.2
237.5	$1M_\odot$	0	11.7	0.94	0.06	30.1	1.0
250	$1M_\odot$	0	11.7	0.96	0.06	30.0	0.71
200	$1M_\odot$	-1.0	13.4	0.94	0	32.4	1.3
200	$1M_\odot$	-0.5	12.6	0.95	0.01	32.2	1.5
200	$1M_\odot$	0.5	10.7	0.94	0.1	28.3	0.91
200	$1M_\odot$	1.0	9.7	0.98	0.41	27.3	0.43
200	$1M_\odot$	1.5	8.8	0.99	0.79	23.0	0.04
200	$1M_\odot$	2.0	7.5	1.00	0.99	—	0
200	$1M_\odot$	2.35	7.4	1.00	1.00	—	0
200	$3M_\odot$	-1.0	13.4	0.85	0.03	33.3	1.5
200	$3M_\odot$	-0.5	12.6	0.94	0.01	31.9	1.3
200	$3M_\odot$	0	11.7	0.95	0.05	30.4	1.5
200	$3M_\odot$	0.5	10.7	0.94	0.11	29.2	1.0
200	$3M_\odot$	1.0	9.7	0.99	0.48	25.3	0.38
200	$3M_\odot$	1.5	8.8	0.99	0.85	24.7	0.04
200	$3M_\odot$	2.0	7.5	1.00	1.00	—	0

Table 1—Continued

$V_{\rm esc}$ (km s^{-1})^b	$M_{\rm ms,max}{}^c$	α^d	$\langle M_{\rm BH}\rangle(M_\odot)^e$	$f_{\rm merge}{}^f$	$f_{\rm notBH}{}^g$	$\langle M_{\rm bin,merge}\rangle(M_\odot)^h$	$\langle N_{\rm single,eject}\rangle^i$
200	$3M_\odot$	2.35	7.4	1.00	1.00	—	0

[a]All runs had 100 realizations.

[b]Escape speed from cluster.

[c]Maximum mass of main sequence star.

[d]Number distribution of stars on zero age main sequence: $dN/dM \propto M^{-\alpha}$.

[e]Average mass of all black holes given α and our evolutionary assumptions.

[f]Fraction of runs in which holes merged rather than being ejected.

[g]Fraction of runs in which holes merged with something other than another black hole.

[h]Average mass of double BH binaries that merged.

[i]Average number of single black holes ejected per binary that merged.

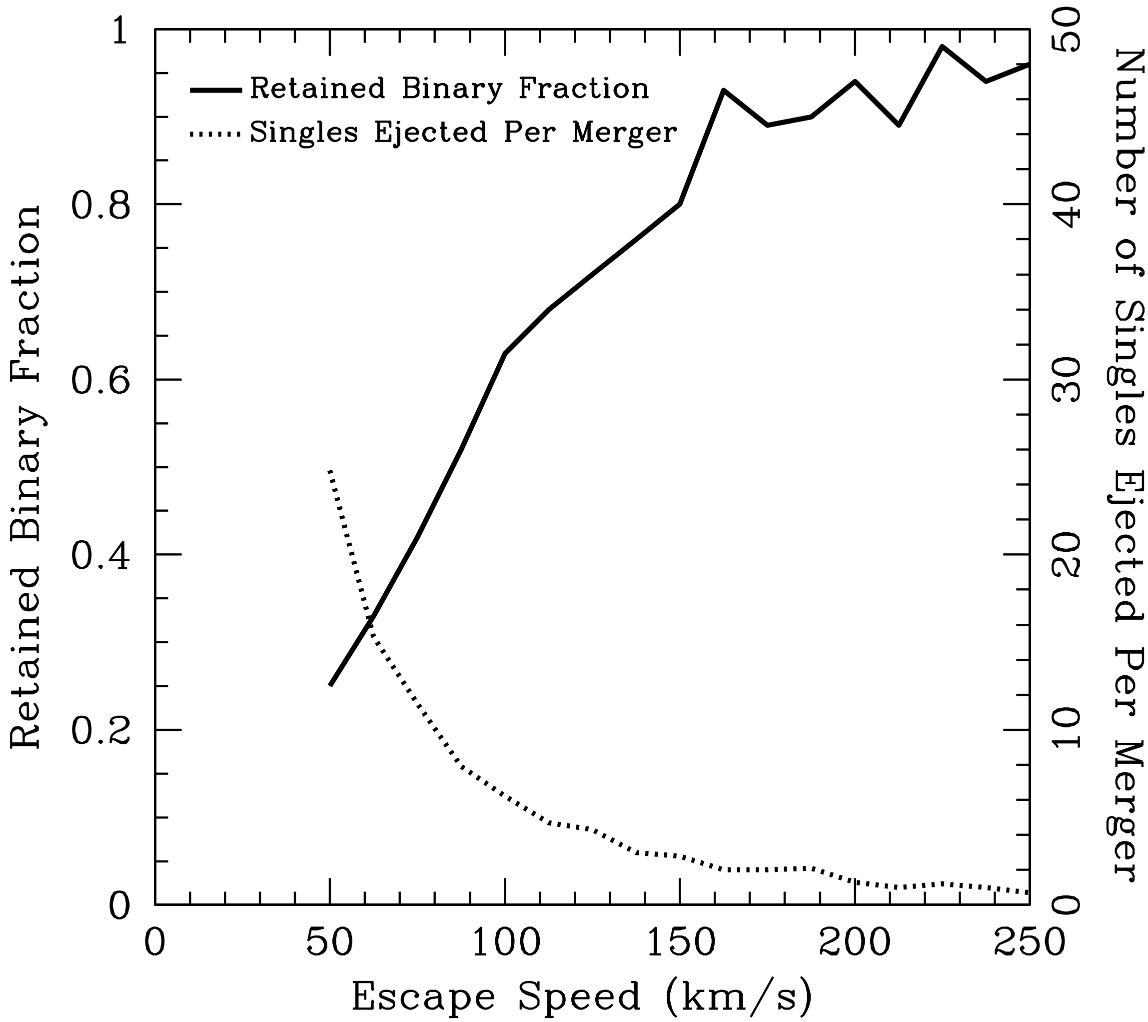

Fig. 1.— Fraction of binaries retained in the nuclear star cluster (solid line) and average number of black holes ejected per black hole merger (dotted line) as a function of the cluster escape speed. Here the zero age main sequence distribution of masses is $dN/dM \propto M^0$, to account for mass segregation in the cluster center, where most interactions occur. We also assume a maximum black hole mass of 20 $M_\odot$ and a maximum main sequence mass of 1 $M_\odot$, but most results are robust against variations of these quantities. All runs are done with 100 realizations, which explains the lack of perfect smoothness. We see, as expected, that the retention fraction increases rapidly with escape speed, so that for nuclear star clusters most binaries stay in the cluster until merger. We also see that at $V_{\rm esc} \sim 200$ km s^{-1} and above, most black hole singles also stay in the cluster. This suggests a high merger efficiency.